# The Alternative Error Model and its Impact on Traceability and Key Comparison

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*go clockwise, start at noon*

## Analysis of Variance

Compare the means $\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_m$ on the basis of

$$x_{il} = x_{0,i} + (x_{il} - \mu_i) + f_i$$

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{m}\sum_{l=1}^{n} x_{il} \qquad N = m \times n$$

$$s^2 = \frac{1}{N-1}\sum_{i=1}^{m}\sum_{l=1}^{n}(x_{il}-\bar{x})^2 \qquad s_1^2 = \frac{n}{m-1}\sum_{i=1}^{m}(\bar{x}_i-\bar{x})^2 \qquad s_2^2 = \frac{1}{N-m}\sum_{i=1}^{m}\sum_{l=1}^{n}(x_{il}-\bar{x}_i)^2$$

$$E\{S_2^2\} = \sigma^2; \quad E\{S_1^2\} = \sigma^2 + \underbrace{\frac{n}{m-1}\sum_{i=1}^{m}\left(f_i - \frac{1}{m}\sum_{j=1}^{m} f_j\right)^2}_{\text{term causing break down}}$$

## The Alternative Error Model

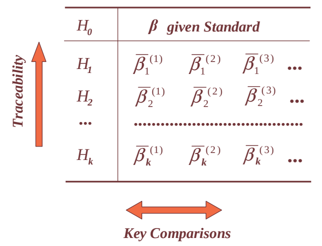
is defined through

(i) stationary experimental set ups,

(ii) the introduction of true values and

(iii) a non-probabilistic treatment of systematic errors

## Hierarchies $H_k$ and Standards $\bar{\beta}_k^{(i)}$




## KCRV as Weighted Average

Averaging m calibrations $\bar{X}_1^{(i)} \pm u_{\bar{X}_1^{(i)}}; \; i = 1,\ldots,m$

$$\bar{\beta} = \sum_{i=1}^{m} w_i \bar{X}_1^{(i)}, \quad w_i = g_i^2 \Big/ \sum_{i=1}^{m} g_i^2, \quad g_i = 1/u_{\bar{X}_1^{(i)}}$$

$$\bar{\beta} = \beta_0 + \sum_{i=1}^{m} w_i(\bar{x}_i - \mu_{\bar{x}_i}) + \sum_{i=1}^{m} w_i f_{\bar{x}_i}$$

uncertainty with respect to the true value: $\beta_0$

$$u_{\bar{\beta}} = \frac{t_P(n-1)}{\sqrt{n}}\sqrt{\sum_{i,j}^{m} w_i w_j s_{ij}} + \sum_{i=1}^{m} w_i f_{s,i}$$

**Traceability and Key Comparisons Should be Referred to the *True Values* of the Physical Quantities Aimed at**

## Model of a Comparator

$$\beta_k^{(i)} = \beta + x$$

$\beta_k^{(i)}$ Standard to be traced back to $\beta$

$\beta$ Primary Standard

$x$ Readings from comparator

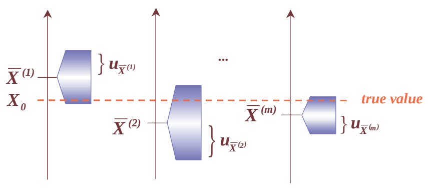
basic error model: $l = 1,\ldots,n$

$$x_l = \underbrace{x_0}_{\text{true value}} + \underbrace{(x_l - \mu_x)}_{\text{random error}} + \underbrace{f_x}_{\text{systematic error}}; \quad -f_{s,x} \le f_x \le f_{s,x}$$

$$\bar{x} - u_{\bar{x}} \le x_0 \le \bar{x} + u_{\bar{x}}; \quad u_{\bar{x}} = \frac{t_P(n-1)}{\sqrt{n}} s_x + f_{s,x}$$

## Key Comparison (iii)

Round Robin




the uncertainties of the participants should localize the true value

## Key Comparison (ii)

Calibrations, $i=1,\ldots,m$

$$\bar{X}^{(i)} = \bar{\beta}_1^{(i)} + \bar{x}_1^{(i)}; \qquad u_{\bar{X}^{(i)}} = \underbrace{u_{\bar{\beta}_1^{(i)}}}_{\text{standard}} + \underbrace{u_{\bar{x}_1^{(i)}}}_{\text{comparator}}$$

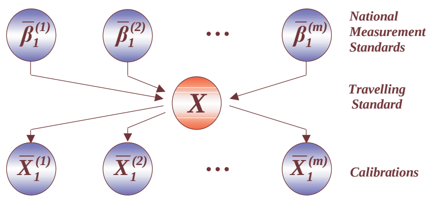
the intervall

$$\bar{X}^{(i)} - u_{\bar{X}^{(i)}} \le X_0 \le \bar{X}^{(i)} - u_{\bar{X}^{(i)}}$$

localizes the true value $X_0$ of the travelling standard

## Key Comparison (i)

Round Robin: m participants




## Literature


**Cox, M.G. and P.M. Harris**, *Towards an objective approach to key comparison reference values, CIE Expert Symposium on Uncertainty Evaluation, Vienna, Austria 22-24 January 2001*

**Grabe, M.**, *On measurement uncertainties derived from „Metrological Statistics“, Algorithms For Approximation IV, University of Huddersfield, U.K. July 15-20, 2001, Proceedings to appear*